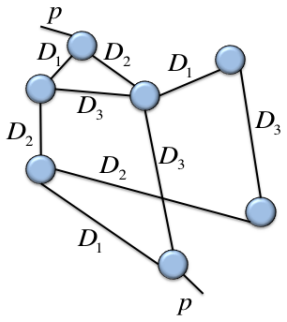
Exercises 3: Tensor Networks and MPS

1.- Tensor Network Contraction

Consider the following tensor network:



Compute the optimal contraction strategy in the case od $p = 2, D_1 = 2, D_2 = 3, D_3 = 4$. What is the asymptotic behavior for general values of the bond dimensions?

2.- 3-point correlator of an infinite MPS

Compute the asymptotic behavior of the 3-point correlation function

$$C(r,s) \equiv \langle O_i O'_{i+r} O''_{i+r+s} \rangle - \langle O_i \rangle \langle O'_{i+r} \rangle \langle O''_{i+r+s} \rangle \tag{1}$$

for $r, s \gg 1$, for an infinite-MPS with translational invariance over one site (i.e. same tensor everywhere) and bond dimension D. What are the relevant length scales?

3.- Correlation length of MPS

Compute the correlation length $\xi = -1/\log|\lambda_2/\lambda_1|$ of the MPS for the GHZ state, 1d cluster state, AKLT state, and Majumdar-Gosh state (these states where explained in class as explicit examples of MPS, and remember that λ_1 and λ_2 are the two largest eigenvalues of the MPS transfer matrix).