Quantum Hall Effect and Topological Order, Exercises 3

1.- Chern insulators: Consider the one-particle Hamiltonian on a square lattice written in momentum space as

$$H(\vec{k}) = (\sin k_x)\sigma_x + (\sin k_y)\sigma_y + (m + \cos k_x + \cos k_y)\sigma_z, \tag{1}$$

where σ_α is the usual Pauli matrix in the direction α. This is a simple *two-band*minimal model.

(a) Compute the eigenvalues of the Hamiltonian as a function of k_x and k_y. For which values of m tis the system gapped?

(b) Let's assume that the first band is completely filled, whereas the second one is empty. Compute the value of the Hall conductivity using the TKNN formula

$$\sigma_{xy} = \frac{e^2}{2\pi\hbar}C, \tag{2}$$

where C is the Chern number of the filled band. How does this Hall conductivity depend on parameter m?

(c) For which values of m and $\vec{k}$ does the gap go to zero? How is the dispersion relation of the excitations around those points? (hint: Taylor-expand the energy around those points in momentum space).